The atom was defined by the ancient Greeks as the smallest particle of matter. We now know that atoms are made up of many smaller particles. Nuclear physicists study these particles using large machines—such as linear accelerators, synchrotrons, and cyclotrons—that can smash atoms to uncover their component parts.

Nuclear physicists use mathematics along with the data they discover to create models of the atom and the structure of matter.
**Vocabulary**

Choose the best term from the list to complete each sentence.

1. According to the __?__, you must multiply or divide before you add or subtract when simplifying a numerical __?__.
2. An algebraic expression is a mathematical sentence that has at least one __?__.
3. In a(n) __?__, an equal sign is used to show that two quantities are the same.
4. You use a(n) __?__ to show that one quantity is greater than another quantity.

Complete these exercises to review skills you will need for this chapter.

**Order of Operations**

Simplify by using the order of operations.

5. \(12 + 4(2)\)  
6. \(12 + 8 \div 4\)  
7. \(15(14 - 4)\)  
8. \((23 - 5) - 36 \div 2\)  
9. \(12 \div 2 + 10 \div 5\)  
10. \(40 \div 2 \cdot 4\)

**Equations**

Solve.

11. \(x + 9 = 21\)  
12. \(3z = 42\)  
13. \(\frac{w}{4} = 16\)  
14. \(24 + t = 24\)  
15. \(p - 7 = 23\)  
16. \(12m = 0\)

**Use Repeated Multiplication**

Find the product.

17. \(7 \times 7 \times 7 \times 7 \times 7\)  
18. \(12 \times 12 \times 12\)  
19. \(3 \times 3 \times 3 \times 3\)  
20. \(11 \times 11 \times 11 \times 11\)  
21. \(8 \times 8 \times 8 \times 8 \times 8\)  
22. \(2 \times 2 \times 2\)  
23. \(100 \times 100 \times 100 \times 100\)  
24. \(9 \times 9 \times 9 \times 9 \times 9\)  
25. \(1 \times 1 \times 1 \times 1\)

**Multiply and Divide by Powers of Ten**

Multiply or divide.

26. \(358(10)\)  
27. \(358(1000)\)  
28. \(358(100,000)\)  
29. \(\frac{358}{10}\)  
30. \(\frac{358}{1000}\)  
31. \(\frac{358}{100,000}\)
Where You’ve Been

Previously, you
• simplified expressions involving order of operations and exponents.
• used models to represent squares and square roots.

In This Chapter

You will study
• expressing numbers in scientific notation, including negative exponents.
• approximating the values of irrational numbers.
• modeling the Pythagorean Theorem.
• using the Pythagorean Theorem to solve real-life problems.

Where You’re Going

You can use the skills learned in this chapter
• to evaluate expressions containing exponents in future math courses.
• to express the magnitude of interstellar distances.
• to use right triangle geometry in future math courses.

Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponent</td>
<td>exponente</td>
</tr>
<tr>
<td>hypotenuse</td>
<td>hipotenusa</td>
</tr>
<tr>
<td>irrational number</td>
<td>número irracional</td>
</tr>
<tr>
<td>perfect square</td>
<td>cuadrado perfecto</td>
</tr>
<tr>
<td>power</td>
<td>potencia</td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>teorema de Pitágoras</td>
</tr>
<tr>
<td>real number</td>
<td>número real</td>
</tr>
<tr>
<td>scientific notation</td>
<td>notación científica</td>
</tr>
</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *irrational* contains the prefix *ir-*, which means “not.” Knowing what you do about rational numbers, what do you think is true of irrational numbers?

2. The word *real* means “actual” or “genuine.” How do you think this applies to math, and how do you think real numbers differ from numbers that are not real?
**Study Strategy: Take Effective Notes**

Good note-taking is an important study strategy. The Cornell system of note taking is an effective way to organize and review main ideas. This method involves dividing your notebook paper into three main sections. You take notes in the note-taking column during the lecture. You write questions and key phrases in the cue column as you review your notes. You write a brief summary of the lecture in the summary area.

**Step 1: Notes**
Draw a vertical line about 2.5 inches from the left side of your paper. During class, write your notes about the main points of the lecture in the right column.

**Step 2: Cues**
After class, write down key phrases or questions in the left column.

**Step 3: Summary**
Use the cues to restate the main points in your own words.

---

**Try This**

1. Research and write a paragraph describing the Cornell system of note taking. Describe how you can benefit from using this type of system.

2. In your next class, use the Cornell system of note taking. Compare these notes to your notes from a previous lecture. Do you think your old notes or the notes using the Cornell system would better prepare you for tests and quizzes?
Learn to evaluate expressions with exponents.

**Vocabulary**
- **exponential form**
- **exponent**
- **base**
- **power**

Fold a piece of $8\frac{1}{2}$-by-11-inch paper in half. If you fold it in half again, the paper is 4 sheets thick. After the third fold in half, the paper is 8 sheets thick. How many sheets thick is the paper after 7 folds?

With each fold the number of sheets doubles.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128 \text{ sheets thick after 7 folds}$$

This multiplication problem can also be written in **exponential form**.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$$  *The number 2 is a factor 7 times.*

If a number is in **exponential form**, the **exponent** represents how many times the **base** is to be used as a factor. A number produced by raising a base to an exponent is called a **power**. Both 27 and 33 represent the same power.

**EXAMPLE 1**
**Writing Exponents**

Write in exponential form.

- **A** $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$
  $$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^7$$  *Identify how many times 5 is a factor.*

- **B** $(−4) \cdot (−4) \cdot (−4)$
  $$(−4) \cdot (−4) \cdot (−4) = (−4)^3$$  *Identify how many times −4 is a factor.*

- **C** $8 \cdot 8 \cdot 8 \cdot 8 \cdot p \cdot p \cdot p$
  $$8 \cdot 8 \cdot 8 \cdot 8 \cdot p \cdot p \cdot p = 8^4p^3$$  *Identify how many times 8 and p are each used as a factor.*

**EXAMPLE 2**
**Evaluating Powers**

Evaluate.

- **A** $3^4$
  $$3^4 = 3 \cdot 3 \cdot 3 \cdot 3$$  *Find the product of four 3’s.*
  $$= 81$$

- **B** $12^2$
  $$12^2 = 12 \cdot 12$$  *Find the product of two 12’s.*
  $$= 144$$
Evaluate.

\[ (-8)^2 \]

\[ (-8)^2 = (-8) \cdot (-8) \quad \text{Find the product of two } -8\text{'s.} \]

\[ = 64 \]

\[ -2^3 \]

\[ -2^3 = -(2 \cdot 2 \cdot 2) \quad \text{Find the product of three } 2\text{'s and then make the answer negative.} \]

\[ = -8 \]

**EXAMPLE 3**

**Using the Order of Operations**

Evaluate \( x - y(z \cdot y^2) \) for \( x = 20, y = 4, \) and \( z = 2.\)

\[
x - y(z \cdot y^2)
\]

\[
20 - 4(2 \cdot 4^2)
\]

\[
= 20 - 4(2 \cdot 16)
\]

\[
= 20 - 4(32)
\]

\[
= 20 - 128
\]

\[
= -108
\]

Substitute 20 for \( x, \) 4 for \( y, \) and 2 for \( z.\)

Evaluate the exponent.

Multiply inside the parentheses.

Multiply from left to right.

Subtract from left to right.

**EXAMPLE 4**

**Geometry Application**

The number of diagonals of an \( n\)-sided figure is \( \frac{1}{2}(n^2 - 3n). \) Use the formula to find the number of diagonals for a 6-sided figure.

\[
\frac{1}{2}(n^2 - 3n)
\]

\[
\frac{1}{2}(6^2 - 3 \cdot 6)
\]

\[
\frac{1}{2}(36 - 18)
\]

\[
\frac{1}{2}(18)
\]

\[
9
\]

Substitute the number of sides for \( n.\)

Simplify inside the parentheses.

Subtract inside the parentheses.

Multiply.

A 6-sided figure has 9 diagonals.

You can verify your answer by sketching the diagonals.

**Think and Discuss**

1. Explain the difference between \((-5)^2\) and \(-5^2.\)

2. Compare \(3 \cdot 2, 3^2,\) and \(2^3.\)

3. Show that \((4 - 11)^2\) is not equal to \(4^2 - 11^2.\)
4-1 Exercises

GUIDED PRACTICE

Write in exponential form.

1. 12
2. 18 · 18
3. \(2b · 2b · 2b · 2b\)
4. \((-3) · (-3)\)

Evaluate.

5. \(2^6\)
6. \((-7)^2\)
7. \((-5)^3\)
8. \(-7^4\)
9. \(8^4\)

Evaluate each expression for the given values of the variables.

10. \(a^5 + 4b\) for \(a = 3\) and \(b = 12\)
11. \(2x^9 - (y + z)\) for \(x = -1\), \(y = 7\), and \(z = -4\)
12. \(s + (t^u - 1)\) for \(s = 13\), \(t = 5\), and \(u = 3\)
13. \(100 - n(p^q - 4)\) for \(n = 10\), \(p = 3\), and \(q = 8\)

The sum of the first \(n\) positive integers is \(\frac{1}{2}(n^2 + n)\). Check the formula for the first 5 positive integers. Then use the formula to find the sum of the first 14 positive integers.

INDEPENDENT PRACTICE

Write in exponential form.

15. \(5 · 5 · 5 · 5 · 5 · 5\)
16. \((-9) · (-9) · (-9)\)
17. \(3d · 3d · 3d\)
18. \(-8\)
19. \((-4) · (-4) · c · c · c\)
20. \(x · x · y\)

Evaluate.

21. \(4^4\)
22. \((-3)^6\)
23. \(8^5\)
24. \(-2^9\)
25. \((-4)^2\)

Evaluate each expression for the given values of the variables.

26. \(b^2\) for \(b = -7\)
27. \(2^c + 3d(g + 2)\) for \(c = 7\), \(d = 5\), and \(g = 1\)
28. \(m + n^p\) for \(m = 12\), \(n = 11\), and \(p = 2\)
29. \(x + y^z\) for \(x = 9\), \(y = 3\), and \(z = 2\)

A circle can be divided by \(n\) lines into a maximum of \(\frac{1}{2}(n^2 + n) + 1\) regions. Use the formula to find the maximum number of regions for 7 lines.

PRACTICE AND PROBLEM SOLVING

Write in exponential form.

31. \((-3) · (-3) · (-3) · (-3)\)
32. \(5h · 5h · 5h\)
33. \(6 · 6 · 6 · 6 · 6 · 6\)
34. \((4)(4)(4)(4)(4)(4)\)
Evaluate.

35. \(5^3\)  
36. \(8^2\)  
37. \((-14)^3\)  
38. \(-4^5\)

Simplify.

39. \(44 - (5 \cdot 4^2)\)  
40. \((4 + 4^4)\)  
41. \((6 - 7^1)\)  
42. \(84 - [8 - (-2)^3]\)

Evaluate each expression for the given value of the variable.

43. \(m(p - n^6)\) for \(m = 2, n = 6, p = 3,\) and \(q = 3\)

44. \(r + (t \cdot s^6)\) for \(r = 42, s = 4, t = 3,\) and \(v = 2\)

45. **Life Science** Bacteria can divide every 20 minutes, so 1 bacterium can multiply to 2 in 20 minutes, 4 in 40 minutes, and so on. How many bacteria will there be in 6 hours? Write your answer using exponents, and then evaluate.

46. **Critical Thinking** For any whole number \(n, 5^n - 1\) is divisible by 4. Verify this for \(n = 4\) and \(n = 6\).

47. **Estimation** A gift shaped like a cube has sides that measure 12.3 cm long. What is the approximate volume of the gift? (Hint: \(V = s^3\))

48. **Choose a Strategy** Place the numbers 1, 2, 3, 4, and 5 in the boxes to make a true statement: 

\[\square \cdot \square^3 = \square^2 - \square\square\]

49. **Write About It** Compare \(10^2\) and \(2^{10}\). For any two numbers, make a conjecture about which usually gives the greater number, using the greater number as the base or as the exponent. Give at least one exception.

50. **Challenge** Write \((4^2)^3\) using a single exponent.

---

**Test Prep and Spiral Review**

51. **Multiple Choice** Which expression has the greatest value?

- \(A\) \(2^5\)  
- \(B\) \(3^4\)  
- \(C\) \(4^3\)  
- \(D\) \(5^2\)

52. **Multiple Choice** The volume of a cube is calculated by using the formula \(V = s^3\), where \(s\) is the length of the sides of the cube. What is the volume of a cube that has sides 8 meters long?

- \(E\) 24 m\(^3\)  
- \(F\) 512 m\(^3\)  
- \(G\) 888 m\(^3\)  
- \(H\) 6561 m\(^3\)

53. **Gridded Response** What is the value of \(5^4\)?

Find each sum. (Lessons 1-4 and 1-5)

54. \(-18 + -65\)  
55. \(-123 + 95\)  
56. \(87 - (-32)\)  
57. \(-74 - (-27)\)

Write each fraction as a decimal. (Lesson 2-1)

58. \(\frac{7}{50}\)  
59. \(\frac{4}{15}\)  
60. \(\frac{3}{8}\)  
61. \(\frac{5}{24}\)

---

**Life Sciences** Most bacteria reproduce by a type of simple cell division known as binary fission. Each species reproduces best at a specific temperature and moisture level.
Learn to evaluate expressions with negative exponents and to evaluate the zero exponent.

The nanoguitar is the smallest guitar in the world. It is no larger than a single cell, at about $10^{-5}$ meters long.

Look for a pattern in the table to extend what you know about exponents to include negative exponents.

### Using a Pattern to Evaluate Negative Exponents

#### Evaluate the powers of 10.

**A** $10^{-4}$

$$10^{-4} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{10,000} = 0.0001$$

*Extend the pattern from the table. Multiply. Write as a decimal.*

**B** $10^{-5}$

$$10^{-5} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{100,000} = 0.00001$$

*Extend the pattern from Example 1A. Multiply. Write as a decimal.*

### NEGATIVE EXONENTS

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any number except 0 with a negative exponent equals its reciprocal with the opposite exponent.</td>
<td>$5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$</td>
<td>$b^{-n} = \left(\frac{1}{b}\right)^n, b \neq 0$</td>
</tr>
</tbody>
</table>
Evaluating Negative Exponents

Evaluate.

A. \((-2)^{-3}\)
   \[\frac{(-2)^{-3}}{(-2)^{3}}\]
   \[= \left(\frac{1}{-2}\right)^3\]
   \[= \frac{1}{-2} \cdot \frac{1}{-2} \cdot \frac{1}{-2}\]
   \[= -\frac{1}{8}\]
   Write the reciprocal; change the sign of the exponent.
   Find the product of three \(\left(\frac{1}{-2}\right)'s\).
   Simplify.

B. \(6^{-4}\)
   \[\frac{6^{-4}}{6^4}\]
   \[= \left(\frac{1}{6}\right)^4\]
   Write the reciprocal; change the sign of the exponent.
   Find the product of four \(\frac{1}{6}'s\).
   Simplify.

Notice from the table on the previous page that \(10^0 = 1\). This is true for any nonzero number to the zero power.

### THE ZERO POWER

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>The zero power of any number except 0 equals 1.</td>
<td>(100^0 = 1)</td>
<td>(a^0 = 1, \text{ if } a \neq 0)</td>
</tr>
<tr>
<td>((-7)^0 = 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the Order of Operations

Evaluate \(2 + (-7)^0 - (4 + 2)^{-2}\).

\[2 + (-7)^0 - (4 + 2)^{-2}\]
\[= 2 + (-7)^0 - 6^{-2}\]
\[= 2 + 1 - \frac{1}{36}\]
\[= \frac{235}{36}\]
Add inside the parentheses.
Evaluate the exponents.
Add and subtract from left to right.

Think and Discuss

1. Express \(\frac{1}{2}\) using a negative exponent.
2. Tell whether an integer raised to a negative exponent can ever be greater than 1. Justify your answer.
4-2 Exercises

**GUIDED PRACTICE**
See Example 1
Evaluate the powers of 10.
1. \(10^{-2}\)  2. \(10^{-7}\)  3. \(10^{-6}\)  4. \(10^{-10}\)

See Example 2
Evaluate.
5. \((2)^{-6}\)  6. \((-3)^{-4}\)  7. \(3^{-3}\)  8. \((-2)^{-5}\)

See Example 3
9. \(4 + 3(4 - 9^0) + 5^{-3}\)  11. \((2 + 2)^{-2} + (1 + 1)^{-4}\)

**INDEPENDENT PRACTICE**
See Example 1
Evaluate the powers of 10.
13. \(10^{-1}\)  14. \(10^{-9}\)  15. \(10^{-8}\)  16. \(10^{-12}\)

See Example 2
Evaluate.
17. \((-4)^{-1}\)  18. \(5^{-2}\)  19. \((-10)^{-4}\)  20. \((-2)^{-6}\)

See Example 3
21. \(128(2 + 6)^{-3} + (4^0 - 3)\)  23. \(12 - (-5)^0 + (3^{-3} + 9^{-2})\)

22. \(3 + (-3)^{-2} - (9 + 7)^0\)

24. \(5^0 + 49(1 + 6)^{-2}\)

**PRACTICE AND PROBLEM SOLVING**
Evaluate.
25. \((18 - 16)^{-5}\)  26. \(25 + (6 \cdot 10^0)\)  27. \((3 \cdot 3)^{-3}\)  28. \((1 - 2^{-2})\)
29. \(3^{-2} \cdot 2^2 \cdot 4^0\)  30. \(10 + 4^3 \cdot 2^{-2}\)
31. \(6^2 - 3^2 + 1^{-1}\)  32. \(16 - [15 - (-2)^{-3}]\)

Evaluate each expression for the given value of the variable.
33. \(2(x^2 + x)\) for \(x = 2.1\)
34. \((4n)^{-2} + n\) for \(n = 3\)
35. \(c^2 + c\) for \(c = \frac{1}{2}\)
36. \(m^{-2} \cdot m^0 \cdot m^2\) for \(m = 9\)

Write each expression as repeated multiplication. Then evaluate the expression.
37. \(11^{-4}\)  38. \(1^{-10}\)  39. \(-6^{-3}\)  40. \((-6)^{-3}\)

41. Make a table with the column headings \(n, n^{-2}\), and \(-2n\). Complete the table for \(n = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4,\) and 5.

42. **Pattern** Describe the following pattern: \((-1)^1 = \_\); \((-1)^{-2} = \_\); \((-1)^{-3} = \_\); \((-1)^{-4} = \_\). Determine what \((-1)^{-100}\) would be. Justify your thinking.

43. **Critical Thinking** Evaluate \(n^1 \cdot n^{-1}\) for \(n = 1, 2,\) and 3. Then make a conjecture what \(n^1 \cdot n^{-1}\) is for any value of \(n\). Explain your reasoning.
44. The sperm whale is the deepest diving whale. It can dive to depths greater than $10^{12}$ nanometers. Evaluate $10^{12}$.

45. Blubber makes up 27% of a blue whale's body weight. Davis found the average weight of blue whales and used it to calculate the average weight of their blubber. He wrote the amount as $2^2 \times 3^3 \times 5 \times 71$ pounds. Evaluate this amount.

46. Most baleen whales migrate an average of $25 \times 125$ km each way. The gray whale has the longest known migration of any mammal, a distance of $2^4 \times 3 \times 125$ km farther each way than the average baleen whale migration. How far does the gray whale migrate each way?

47. A blue whale may eat between 6 and 7 tons of krill each day. Krill are approximately $2^{-5} \times 3^{-1} \times 5^{-1}$ of the length of a blue whale. Evaluate this amount.

48. **Challenge** A cubic centimeter is the same as 1 mL. If a humpback whale has more than 1 kL of blood, how many cubic centimeters of blood does the humpback whale have?

49. **Multiple Choice** Evaluate $(-5)^{-2}$.

   - A) -25
   - B) $\frac{1}{25}$
   - C) $\frac{1}{25}$
   - D) 25

50. **Extended Response** Evaluate $8^3, 8^2, 8^1, 8^0, 8^{-1}$, and $8^{-2}$. Describe the pattern of the values. Use the pattern of the values to predict the value of $8^{-3}$.

Give the coordinates and quadrant of each point. (Lesson 3-2)

51. $A$  
52. $B$  
53. $C$  
54. $D$

Evaluate. (Lesson 4-1)

55. $(-3)^4$  
56. $5^2$  
57. $(10 - 15)^3$  
58. $(-9)^3$
Learn to apply the properties of exponents.

The factors of a power, such as $7^4$, can be grouped in different ways. Notice the relationship of the exponents in each product.

$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$

$(7 \cdot 7 \cdot 7) \cdot 7 = 7^3 \cdot 7^1 = 7^4$

$(7 \cdot 7) \cdot (7 \cdot 7) = 7^2 \cdot 7^2 = 7^4$

### Multiplying Powers with the Same Base

<table>
<thead>
<tr>
<th>Words</th>
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<th>Algebra</th>
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</thead>
<tbody>
<tr>
<td>To multiply powers with the same base, keep the base and add the exponents.</td>
<td>$3^5 \cdot 3^8 = 3^{5+8} = 3^{13}$</td>
<td>$b^m \cdot b^n = b^{m+n}$</td>
</tr>
</tbody>
</table>

### Example 1

#### Multiplying Powers with the Same Base

Multiply. Write the product as one power.

**A** $5^4 \cdot 5^3$

- $5^4 = 5^3 = 5^7$
- Add exponents.

**B** $a^{12} \cdot a^{12}$

- $a^{12} + 12$
- Add exponents.

**C** $16 \cdot 16^{-7}$

- $16 \cdot 16^{-7}$
- $16^{-7} = 16^1$
- Think: $16 = 16^1$
- Add exponents.

**D** $4^2 \cdot 2^2$

- $4^2 \cdot 2^2$
- Cannot combine; the bases are not the same.

Notice what occurs when you divide powers with the same base.

$$\frac{5^5}{5^3} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5} = \frac{5 \cdot 5}{5} = 5 \cdot 5 = 5^2$$

### Dividing Powers with the Same Base

<table>
<thead>
<tr>
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<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>To divide powers with the same base, keep the base and subtract the exponents.</td>
<td>$6^9 \div 6^4 = 6^{9-4} = 6^5$</td>
<td>$\frac{b^m}{b^n} = b^{m-n}$</td>
</tr>
</tbody>
</table>
Dividing Powers with the Same Base

Divide. Write the quotient as one power.

A. \[
\frac{10^8}{10^5} = 10^{8-5} = 10^3
\]

B. \[
\frac{x^9}{y^4} = x^9 \cdot y^{-4}
\]

Cannot combine; the bases are not the same.

To see what happens when you raise a power to a power, use the order of operations.

\[
(4^3)^2 = (4 \cdot 4 \cdot 4)^2
\]

Evaluate the power inside the parentheses.

\[
= (4 \cdot 4 \cdot 4) \cdot (4 \cdot 4 \cdot 4)
\]

Evaluate the power outside the parentheses.

\[
= 4^6
\]

RAISING A POWER TO A POWER

To raise a power to a power, keep the base and multiply the exponents.

\[
(9^4)^5 = 9^{4 \cdot 5} = 9^{20}
\]

\[
(b^m)^n = b^{m \cdot n}
\]

Raising a Power to a Power

Simplify.

A. \[
(7^5)^3 = 7^{5 \cdot 3} = 7^{15}
\]

Multiply exponents.

B. \[
(8^9)^{11} = 8^{9 \cdot 11} = 8^{99}
\]

Multiply exponents.

C. \[
(2^{-7})^{-2} = 2^{-7 \cdot (-2)} = 2^{14}
\]

Multiply exponents.

D. \[
(12^{10})^{-6} = 12^{10 \cdot (-6)} = 12^{-60}
\]

Multiply exponents.

Think and Discuss

1. Explain why the exponents cannot be added in the product \(14^3 \cdot 18^3\).

2. List two ways to express \(4^5\) as a product of powers.
4-3 Exercises

**GUIDED PRACTICE**

See Example 1 Multiply. Write the product as one power.
1. \(5^6 \cdot 5^9\)  
2. \(12^3 \cdot 12^{-2}\)  
3. \(m \cdot m^3\)  
4. \(5^3 \cdot 7^3\)

See Example 2 Divide. Write the quotient as one power.
5. \(\frac{6^5}{6^3}\)  
6. \(\frac{a^8}{a^{-1}}\)  
7. \(\frac{12^5}{12^3}\)  
8. \(\frac{5^{16}}{5^4}\)

See Example 3 Simplify.
9. \((3^4)^5\)  
10. \((2^2)^0\)  
11. \((4^{-2})^3\)  
12. \((-y^2)^6\)

**INDEPENDENT PRACTICE**

See Example 1 Multiply. Write the product as one power.
13. \(10^{10} \cdot 10^7\)  
14. \(3^4 \cdot 3^4\)  
15. \(r^3 \cdot r^{-2}\)  
16. \(18 \cdot 18^5\)

See Example 2 Divide. Write the quotient as one power.
17. \(\frac{5^{10}}{5^6}\)  
18. \(\frac{m^{10}}{d^3}\)  
19. \(\frac{r^9}{t^{-4}}\)  
20. \(\frac{3^7}{3^7}\)

See Example 3 Simplify.
21. \((5^0)^8\)  
22. \((6^4)^{-1}\)  
23. \((3^{-2})^2\)  
24. \((x^5)^2\)

**PRACTICE AND PROBLEM SOLVING**

Simplify. Write the product or quotient as one power.
25. \(\frac{4^7}{4^3}\)  
26. \(3^8 \cdot 3^{-1}\)  
27. \(\frac{a^4}{a^{-3}}\)  
28. \(\frac{10^{18}}{10^9}\)

29. \(x^3 \cdot x^7\)  
30. \(a^6 \cdot b^9\)  
31. \((7^4)^3\)  
32. \(2 \cdot 2^4\)

33. \(\frac{10^4}{5^2}\)  
34. \(\frac{11^7}{11^6}\)  
35. \(\frac{y^8}{y^8}\)  
36. \(y^8 \cdot y^{-8}\)

37. There are \(26^3\) ways to make a 3-letter “word” (from aaa to zzz) and \(26^5\) ways to make a 5-letter word. How many times more ways are there to make a 5-letter word than a 3-letter word?

38. **Astronomy** The mass of the sun is about \(10^{27}\) metric tons, or \(10^{30}\) kilograms. How many kilograms are in one metric ton?

39. **Business** Using the manufacturing terms below, tell how many dozen are in a great gross. How many gross are in a great gross?

<table>
<thead>
<tr>
<th>1 dozen</th>
<th>= 12^1 items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gross</td>
<td>= 12^2 items</td>
</tr>
<tr>
<td>1 great gross</td>
<td>= 12^3 items</td>
</tr>
</tbody>
</table>
40. **Estimation** The distance from Earth to the moon is about 22,000 miles. The distance from Earth to Neptune is about 22,000,000 miles. Which distance is greater? About how many times as great?

Find the missing exponent.

41. \(b^5 \cdot b^4 = b^8\)  
42. \((v^2)^3 = v^6\)  
43. \(w^4 \div w^3 = w\)  
44. \((a^4)^3 = a^{12}\)

45. A googol is the number 1 followed by 100 zeros.
   a. What is a googol written as a power?
   b. What is a googol times a googol written as a power?

46. **What's the Error?** A student said that \(\frac{\frac{3}{5}}{\frac{1}{3}}\) is the same as \(\frac{1}{3}\). What mistake has the student made?

47. **Write About It** Why do you subtract exponents when dividing powers with the same base?

48. **Challenge** A number to the 11th power divided by the same number to the 8th power equals 64. What is the number?

### Test Prep and Spiral Review

49. **Multiple Choice** In computer technology, a kilobyte is \(2^{10}\) bytes in size. A gigabyte is \(2^{30}\) bytes in size. The size of a terabyte is the product of the size of a kilobyte and the size of a gigabyte. What is the size of a terabyte?
   - A) \(2^{20}\) bytes
   - B) \(2^{40}\) bytes
   - C) \(2^{300}\) bytes
   - D) \(4^{300}\) bytes

50. **Short Response** A student claims that \(10^3 \cdot 10^{-5}\) is greater than 1. Explain whether the student is correct.

Evaluate each expression for the given value of the variable. (Lesson 2-3)

51. \(19.4 - x\) for \(x = -5.6\)  
52. \(11 - r\) for \(r = 13.5\)  
53. \(p + 65.1\) for \(p = -42.3\)

54. \(-\frac{3}{7} - t\) for \(t = \frac{15}{7}\)  
55. \(3\frac{5}{11} + y\) for \(y = -2\frac{4}{11}\)  
56. \(-\frac{1}{19} + g\) for \(g = \frac{18}{19}\)

Evaluate. (Lesson 4-2)

57. \((-3)^{-2}\)  
58. \((-2)^{-3}\)  
59. \(1^{-3}\)  
60. \((-2)^{-4}\)
Learn to express large and small numbers in scientific notation and to compare two numbers written in scientific notation.

Vocabulary

**scientific notation**

An ordinary quarter contains about 97,700,000,000,000,000,000,000 atoms. The average size of an atom is about 0.00000003 centimeter across.

The length of these numbers in standard notation makes them awkward to work with. **Scientific notation** is a shorthand way of writing such numbers.

To express any number in scientific notation, write it as the product of a power of ten and a number greater than or equal to 1 but less than 10.

In scientific notation, the number of atoms in a quarter is $9.77 \times 10^{22}$, and the size of each atom is $3.0 \times 10^{-8}$ centimeters across.

### Example 1

**Translating Scientific Notation to Standard Notation**

Write each number in standard notation.

**A**

1. $3.12 \times 10^9$
   - $10^9 = 1,000,000,000$
   - Think: Move the decimal right 9 places.
   - $3.12 \times 10^9 = 3,120,000,000$

2. $3.12 \times 10^3$
   - Think: Move the decimal right 3 places.
   - $3.12 \times 10^3 = 3,120,000$

3. $3.12 \times 10^{10}$
   - Think: Move the decimal right 10 places.
   - $3.12 \times 10^{10} = 3,120,000,000,000$

4. $3.12 \times 10^{12}$
   - Think: Move the decimal right 12 places.
   - $3.12 \times 10^{12} = 3,120,000,000,000,000$

**B**

1. $1.35 \times 10^{-4}$
   - $10^{-4} = \frac{1}{10,000}$
   - Divide by the reciprocal.
   - $1.35 \times 10^{-4} = 0.000135$

2. $1.35 \times 10^{-4}$
   - $1.35 \times 10^{-4} = \frac{1}{10,000}$
   - Divide by the reciprocal.
   - $1.35 \times 10^{-4} = 0.000135$

3. $1.35 \div 10,000$
   - Think: Move the decimal left 4 places.
   - $1.35 \div 10,000 = 0.000135$

4. $1.35 \times \frac{1}{10,000}$
   - Think: Move the decimal left 4 places.
   - $1.35 \times \frac{1}{10,000} = 0.000135$

**C**

1. $4.7 \times 10^{-7}$
   - $10^{-7} = \frac{1}{10,000,000}$
   - Divide by the reciprocal.
   - $4.7 \times 10^{-7} = 0.00000047$

2. $4.7 \times 10^{-7}$
   - $4.7 \times 10^{-7} = \frac{1}{10,000,000}$
   - Divide by the reciprocal.
   - $4.7 \times 10^{-7} = 0.00000047$

3. $4.7 \div 10,000,000$
   - Think: Move the decimal left 7 places.
   - $4.7 \div 10,000,000 = 0.00000047$

4. $4.7 \times \frac{1}{10,000,000}$
   - Think: Move the decimal left 7 places.
   - $4.7 \times \frac{1}{10,000,000} = 0.00000047$
Translating Standard Notation to Scientific Notation

Write 0.0000003 in scientific notation.

\[ 0.0000003 \]

Think: The decimal needs to move 7 places to get a number between 1 and 10.

\[ 3 \times 10^{-7} \]

Set up scientific notation.

Think: The decimal needs to move left to change 3 to 0.0000003, so the exponent will be negative.

So 0.0000003 written in scientific notation is \( 3 \times 10^{-7} \).

Check \( 3 \times 10^{-7} = 3 \times 0.0000001 = 0.0000003 \)

Money Application

Suppose you have a million dollars in pennies. A penny is 1.55 mm thick. How tall would a stack of all your pennies be? Write the answer in scientific notation.

\[ \$1.00 = 100 \text{ pennies} \]
\[ \$1,000,000 = 100,000,000 \text{ pennies} \]
\[ 1.55 \text{ mm} \times 100,000,000 \]
\[ 155,000,000 \text{ mm} \]
\[ 1.55 \times 10^{8} \]

Multiply each side by 1,000,000.

Find the total height.

Multiply.

Set up scientific notation.

Think: The decimal needs to move 8 places.

Think: The decimal needs to move right to change 1.55 to 155,000,000, so the exponent will be positive.

In scientific notation the total height of one million dollars in stacked pennies is \( 1.55 \times 10^{8} \) mm. This is about 96 miles tall.

To compare two numbers written in scientific notation, first compare the powers of ten. The number with the greater power of ten is greater. If the powers of ten are the same, compare the values between one and ten.

\[ 2.7 \times 10^{13} > 2.7 \times 10^{9} \]
\[ 10^{13} > 10^{9} \]
\[ 3.98 \times 10^{22} > 2.52 \times 10^{22} \]
\[ 3.98 > 2.52 \]
Life Science Application

The major components of human blood are red blood cells, white blood cells, platelets, and plasma. A typical red blood cell has a diameter of approximately $7 \times 10^{-6}$ meter. A typical platelet has a diameter of approximately $2.33 \times 10^{-6}$ meter. Which has a greater diameter, a red blood cell or a platelet?

$7 \times 10^{-6} \quad 2.33 \times 10^{-6}$

Compare powers of 10.

$10^{-6} = 10^{-6}$

Compare the values between 1 and 10.

$7 > 2.33$

$7 \times 10^{-6} > 2.33 \times 10^{-6}$

A typical red blood cell has a greater diameter than a typical platelet.

Think and Discuss

1. **Explain** the benefit of writing numbers in scientific notation.

2. **Describe** how to write $2.977 \times 10^6$ in standard notation.

3. **Determine** which measurement would be least likely to be written in scientific notation: size of bacteria, speed of a car, or number of stars in a galaxy.

4-4 Exercises

**Guided Practice**

See Example 1 Write each number in standard notation.

1. $4.17 \times 10^3$
2. $1.33 \times 10^{-5}$
3. $6.2 \times 10^7$
4. $3.9 \times 10^{-4}$

See Example 2 Write each number in scientific notation.

5. $0.000057$
6. $0.0004$
7. $6,980,000$
8. $0.000000025$

See Example 3

9. The distance from Earth to the Moon is about 384,000 km. Suppose an astronaut travels this distance a total of 250 times. How many kilometers does the astronaut travel? Write the answer in scientific notation.

See Example 4

10. The maximum length of a particle that can fit through a surgical mask is $1 \times 10^{-4}$ millimeters. The average length of a dust mite is approximately $1.25 \times 10^{-1}$ millimeters. Which is longer, the largest particle that can fit through a surgical mask or a dust mite of average length?
INDEPENDENT PRACTICE

Write each number in standard notation.

11. $9.2 \times 10^6$
12. $6.7 \times 10^4$
13. $3.6 \times 10^{-2}$
14. $5.24 \times 10^8$

Write each number in scientific notation.

15. $0.00007$
16. $6,500,000$
17. $100,000,000$
18. $0.00000003$

19. Protons and neutrons are the most massive particles in the nucleus of an atom. If a nucleus were the size of an average grape, it would have a mass greater than 9 million metric tons. A metric ton is 1000 kg. What would the mass of a grape-size nucleus be in kilograms? Write your answer in scientific notation.

20. The orbits of Neptune and Pluto cross each other. Neptune’s average distance from the Sun is approximately $4.5 \times 10^9$ kilometers. Pluto’s average distance from the Sun is approximately $5.87 \times 10^9$ kilometers. Which planet has the greater average distance from the Sun?

Write each number in standard notation.

21. $1.4 \times 10^5$
22. $3.24 \times 10^{-2}$
23. $7.8 \times 10^1$
24. $2.1 \times 10^{-6}$

25. $5.3 \times 10^{-8}$
26. $8.456 \times 10^{-4}$
27. $5.59 \times 10^5$
28. $7.1 \times 10^3$

29. $7.113 \times 10^6$
30. $4.5 \times 10^{-1}$
31. $2.9 \times 10^{-4}$
32. $5.6 \times 10^2$

33. Life Science Duckweed plants live on the surface of calm ponds and are the smallest flowering plants in the world. They weigh about 0.00015 g.
   a. Write this number in scientific notation.
   b. If left unchecked, one duckweed plant, which reproduces every 30–36 hours, could produce $1 \times 10^{30}$ (a nonillion) plants in four months. How much would one nonillion duckweed plants weigh?

34. Life Science The diameter of a human red blood cell ranges from approximately $6 \times 10^{-6}$ to $8 \times 10^{-6}$ meters. Write this range in standard notation.

35. Physical Science The atomic mass of an element is the mass, in grams, of one mole (mol), or $6.02 \times 10^{23}$ atoms.
   a. How many atoms are there in 2.5 mol of helium?
   b. If you know that 2.5 mol of helium weighs 10 grams, what is the atomic mass of helium?
   c. Using your answer from part b, find the approximate mass of one atom of helium.
36. **Social Studies**
   a. Express the population and area of Taiwan in scientific notation.
   b. Divide the number of square miles by the population to find the number of square miles per person in Taiwan. Express your answer in scientific notation.

Write each number in scientific notation.
37. 0.00858 38. 0.00000063 39. 5,900,000
40. 7,045,000,000 41. 0.0076 42. 400
43. 4200 44. 0.0000000082 45. 0.0000000003
46. 0.000005 47. 7,000,000 48. 0.0095678

49. Order the list of numbers below from least to greatest.
   $1.5 \times 10^{-2}$, $1.2 \times 10^{6}$, $5.85 \times 10^{-3}$, $2.3 \times 10^{-2}$, $5.5 \times 10^{6}$

50. **Write a Problem** An electron has a mass of about $9.11 \times 10^{-31}$ kg. Use this information to write a problem.

51. **Write About It** Two numbers are written in scientific notation. How can you tell which number is greater?

52. **Challenge** Where on a number line does the value of a positive number in scientific notation with a negative exponent lie?

### Test Prep and Spiral Review

53. **Short Response** Explain how you can determine the sign of the exponent when 29,600,000,000,000 is written in scientific notation.

54. **Multiple Choice** The distance light can travel in one year is $9.46 \times 10^{12}$ kilometers. What is this distance in standard form?
   
   A. $94,600,000,000,000$ km  
   B. $946,000,000,000$ km  
   C. $9,460,000,000,000$ km  
   D. $0.0000000000946$ km

Use each table to make a graph and to write an equation. (Lesson 3-5)

55. | x  | 0 | 5 | 6 | 8 |
    |---|---|---|---|
    | y | -4 | 11 | 14 | 20 |

56. | x  | 0 | 1 | 3 | 6 |
    |---|---|---|---|
    | y | 6 | 7 | 9 | 12 |

Simplify. Write each product or quotient as one power. (Lesson 4-3)

57. $\frac{7^4}{7^2}$  
58. $5^3 \cdot 5^8$  
59. $\frac{t^8}{t^5}$  
60. $10^9 \cdot 10^{-3}$
Multiply and Divide Numbers in Scientific Notation

You can use a graphing calculator to perform operations with numbers written in scientific notation. Use the key combination \( \text{2nd} \, \text{EE} \) to enter numbers in scientific notation. On a graphing calculator, \( 9.5 \times 10^{16} \) is displayed as \( 9.5 \times 10^{16} \).

**Activity**

Use a calculator to find \((4.8 \times 10^{12})(9.4 \times 10^9)\).

Press \( 4.8 \, \text{2nd} \, \text{EE} \, 12 \, \times \, 9.4 \, \text{2nd} \, \text{EE} \, 9 \, \text{ENTER} \).

The calculator displays the answer \( 4.512 \times 10^{22} \), which is the same as \( 4.512 \times 10^{22} \).

**Think and Discuss**

1. When you use the associative and communicative properties to multiply \( 4.8 \times 10^{12} \) and \( 9.4 \times 10^9 \), you get \((4.8 \cdot 9.4)(10^{12} \cdot 10^9) = 45.12 \times 10^{21} \). Explain why this answer is different from the answer you obtained in the activity.

**Try This**

Use a graphing calculator to evaluate each expression.

1. \((5.76 \times 10^{13})(6.23 \times 10^{-20})\)

4. \(\frac{5.25 \times 10^{13}}{6.14 \times 10^8}\)

7. \((2.74 \times 10^{11})(3.2 \times 10^{-5})\)

2. \(\frac{9.7 \times 10^{16}}{2.9 \times 10^2}\)

5. \((1.1 \times 10^9)(2.2 \times 10^3)\)

8. \(\frac{5.82 \times 10^{-11}}{8.96 \times 10^{11}}\)

3. \((1.6 \times 10^5)(9.65 \times 10^9)\)

6. \(\frac{8.56 \times 10^9}{2.34 \times 10^{80}}\)

9. \((4.5 \times 10^{12})(3.7 \times 10^9)\)

10. The star Betelgeuse, in the constellation of Orion, is approximately \( 3.36 \times 10^{15} \) miles from Earth. This is approximately \( 1.24 \times 10^6 \) times as far as Pluto’s minimum distance from Earth. What is Pluto’s approximate minimum distance from Earth? Write your answer in scientific notation.

11. If 446 billion telephone calls were placed by 135 million United States telephone subscribers, what was the average number of calls placed per subscriber?
Quiz for Lessons 4-1 Through 4-4

4-1 Exponents

Evaluate.
1. \(10^1\)
2. \(8^6\)
3. \(-3^4\)
4. \((-5)^3\)
5. Write \(5 \cdot 5 \cdot 5 \cdot 5\) in exponential form.
6. Evaluate \(a^7 - 4b\) for \(a = 3\) and \(b = -1\).

4-2 Look for a Pattern in Integer Exponents

Evaluate.
7. \(10^{-6}\)
8. \((-3)^{-4}\)
9. \(-6^{-2}\)
10. \(4^0\)
11. \(8 + 10^0(-6)\)
12. \(5^{-1} + 3(5)^{-2}\)
13. \(-4^{-3} + 2^0\)
14. \(3^{-2} - (6^0 - 6^{-2})\)

4-3 Properties of Exponents

Simplify. Write the product or quotient as one power.
15. \(9^3 \cdot 9^5\)
16. \(\frac{5^{10}}{5^{10}}\)
17. \(q^9 \cdot q^6\)
18. \(3^3 \cdot 3^{-2}\)

Simplify.
19. \((3^3)^{-2}\)
20. \((4^2)^0\)
21. \((-x^2)^4\)
22. \((4^{-2})^5\)

23. The mass of the known universe is about \(10^{23}\) solar masses, which is \(10^{50}\) metric tons. How many metric tons is one solar mass?

4-4 Scientific Notation

Write each number in scientific notation.
24. \(0.00000015\)
25. \(99,980,000\)
26. \(0.434\)
27. \(100\)

Write each number in standard notation.
28. \(1.38 \times 10^5\)
29. \(4 \times 10^6\)
30. \(1.2 \times 10^{-3}\)
31. \(9.37 \times 10^{-5}\)

32. The average distance from Earth to the Sun is approximately 149,600,000 kilometers. Pluto is about 39.5 times as far from the Sun as Earth is. What is the approximate average distance from Pluto to the Sun? Write your answer in scientific notation.

33. Picoplankton can be as small as 0.00002 centimeter. Microplankton are about 100 times as large as picoplankton. How large is a microplankton that is 100 times the size of the smallest picoplankton? Write your answer in scientific notation.
Mary is making a string of beads. If each bead is $7.0 \times 10^{-1}$ cm wide, how many beads does she need to make a string that is 35 cm long?

The total area of the United States is $9.63 \times 10^6$ square kilometers. The total area of Canada is $9.98 \times 10^6$ square kilometers. What is the total area of both the United States and Canada?

Suppose $\frac{1}{3}$ of the fish in a lake are considered game fish. Of these, $\frac{2}{5}$ meet the legal minimum size requirement. What fraction of the fish in the lake are game fish that meet the legal minimum size requirement?

Part of a checkbook register is shown below. Find the amount in the account after the transactions shown.

<table>
<thead>
<tr>
<th>TRANSACTION</th>
<th>DATE</th>
<th>DESCRIPTION</th>
<th>AMOUNT</th>
<th>FEE</th>
<th>DEPOSITS</th>
<th>BALANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>withdrawal</td>
<td>11/16</td>
<td>autodebit for phone bill</td>
<td>$43.16</td>
<td></td>
<td></td>
<td>$287.34</td>
</tr>
<tr>
<td>Check 1256</td>
<td>11/18</td>
<td>groceries</td>
<td>$27.56</td>
<td></td>
<td></td>
<td>$27.56</td>
</tr>
<tr>
<td>Check 1257</td>
<td>11/23</td>
<td>new clothes</td>
<td>$74.23</td>
<td></td>
<td></td>
<td>$74.23</td>
</tr>
<tr>
<td>withdrawal</td>
<td>11/27</td>
<td>ATM withdrawal</td>
<td>$40.00</td>
<td>$1.25</td>
<td></td>
<td>$41.25</td>
</tr>
</tbody>
</table>
Learn to find square roots.

Vocabulary
principal square root
perfect square

Think about the relationship between the area of a square and the length of one of its sides.

\[
\text{area} = 36 \text{ square units} \\
\text{side length} = \sqrt{36} = 6 \text{ units}
\]

Taking the square root of a number is the inverse of squaring the number.

\[
6^2 = 36 \quad \sqrt{36} = 6
\]

Every positive number has two square roots, one positive and one negative. One square root of 16 is 4, since \(4 \cdot 4 = 16\). The other square root of 16 is \(-4\), since \((-4)(-4) = 16\). You can write the square roots of 16 as \(\pm 4\), meaning “plus or minus” 4.

When you press the \(\sqrt{\phantom{x}}\) key on a calculator, only the nonnegative square root appears. This is called the principal square root of the number.

\[
+\sqrt{16} = 4 \quad -\sqrt{16} = -4
\]

The numbers 16, 36, and 49 are examples of perfect squares. A perfect square is a number that has integers as its square roots. Other perfect squares include 1, 4, 9, 25, 64, and 81.

**Example 1** Finding the Positive and Negative Square Roots of a Number

Find the two square roots of each number.

**A** 81

\[
\sqrt{81} = 9 \\
-\sqrt{81} = -9
\]

9 is a square root, since \(9 \cdot 9 = 81\).

-9 is also a square root, since \(-9 \cdot -9 = 81\).

**B** 1

\[
\sqrt{1} = 1 \\
-\sqrt{1} = -1
\]

1 is a square root, since \(1 \cdot 1 = 1\).

-1 is also a square root, since \(-1 \cdot -1 = 1\).

**C** 144

\[
\sqrt{144} = 12 \\
-\sqrt{144} = -12
\]

12 is a square root, since \(12 \cdot 12 = 144\).

-12 is also a square root, since \(-12 \cdot (-12) = 144\).
**Example 2**

**Computer Application**

The square computer icon contains 676 pixels. How many pixels tall is the icon?

Find the square root of 676 to find the length of the side. Use the positive square root; a negative length has no meaning.

\[26^2 = 676\]

So \(\sqrt{676} = 26\).

The icon is 26 pixels tall.

In the order of operations everything under the square root symbol is treated as if it were in parentheses. \(\sqrt{5 - 3} = \sqrt{5 - 3}\)

**Example 3**

**Evaluating Expressions Involving Square Roots**

Evaluate each expression.

A. \(3\sqrt{25} + 4\)

\[3\sqrt{25} + 4 = 3(5) + 4\]

Evaluate the square root.

\[= 15 + 4\]

Multiply.

\[= 19\]

Add.

B. \(\frac{\sqrt{16}}{4} + \frac{1}{2}\)

\[\frac{\sqrt{16}}{4} + \frac{1}{2} = \frac{4}{4} + \frac{1}{2}\]

Evaluate the square roots.

\[= 2 + \frac{1}{2}\]

\[= \frac{5}{2}\]

Add.

**Think and Discuss**

1. **Describe** what is meant by a perfect square. Give an example.

2. **Explain** how many square roots a positive number can have. How are these square roots different?

3. **Decide** how many square roots 0 has. Tell what you know about square roots of negative numbers.
4-5 Exercises

GUIDED PRACTICE

Find the two square roots of each number.
1. 4  2. 16  3. 64  4. 121
5. 1  6. 441  7. 9  8. 484

9. A square court for playing the game four square has an area of 256 ft². How long is one side of the court?

Evaluate each expression.
10. \( \sqrt{5} + 11 \)
11. \( \frac{81}{9} \)
12. \( 3\sqrt{400} - 125 \)
13. \(-\left(\sqrt{169} - \sqrt{144}\right)\)

INDEPENDENT PRACTICE

Find the two square roots of each number.
14. 25  15. 144  16. 81  17. 169
18. 196  19. 400  20. 361  21. 225

22. Elisa found a square digital image of a famous painting on a Web site. The image contained 360,000 pixels. How many pixels high is the image?

Evaluate each expression.
23. \( \sqrt{25} - 6 \)
24. \( \frac{64}{4} \)
25. \(-\left(\sqrt{36}\sqrt{9}\right)\)
26. \( 5(\sqrt{225} - 10) \)

PRACTICE AND PROBLEM SOLVING

Find the two square roots of each number.
27. 36  28. 100  29. 49  30. 900
31. 529  32. 289  33. 576  34. 324

35. Estimation Mr. Barada bought a square rug. The area of the rug was about 68.06 ft². He estimated that the length of a side was about 7 ft. Is Mr. Barada’s estimate reasonable? Explain.

36. Language Arts Crelle’s Journal is the oldest mathematics periodical in existence. Zacharias Dase’s incredible calculating skills were made famous by Crelle’s Journal in 1844. Dase produced a table of factors of all numbers between 7,000,000 and 10,000,000. He listed 7,022,500 as a perfect square. What is the square root of 7,022,500?

37. Sports A karate match is held on a square mat that has an area of 676 ft². What is the length of the mat?
Find the two square roots of each number.

38. \(\frac{1}{9}\)  
39. \(\frac{1}{121}\)  
40. \(\frac{16}{9}\)  
41. \(\frac{81}{16}\)
42. \(\frac{9}{4}\)  
43. \(\frac{324}{81}\)  
44. \(\frac{1000}{100,000}\)  
45. \(\frac{169}{676}\)

46. **Multi-Step** An office building has a square courtyard with an area of 289 ft\(^2\). What is the distance around the edge of the courtyard?

47. **Games** A chessboard contains 32 black and 32 white squares. How many squares are along each side of the game board?

48. **Hobbies** A quilter wants to use as many of his 65 small fabric squares as possible to make one large square quilt.
   
a. How many small squares can the quilter use? How many small squares would he have left?
   
b. How many more small squares would the quilter need to make the next largest possible square quilt?

49. **What’s the Error?** A student said that since the square roots of a certain number are 1.5 and \(-1.5\), the number must be their product, \(-2.25\). What error did the student make?

50. **Write About It** Explain the steps you would take to evaluate the expression \(\sqrt{14} + 35 - 20\).

51. **Challenge** The square root of a number is four less than three times seven. What is the number?

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**Test Prep and Spiral Review**

52. **Multiple Choice** Which number does NOT have a square root that is an integer?

(A) 81  
(B) 196  
(C) 288  
(D) 400

53. **Short Response** Deanna knows that the floor in her kitchen is a square with an area of 169 square feet. The perimeter of her kitchen floor is found by adding the lengths of all its sides. What is the perimeter of her kitchen floor? Explain your answer.

Write each decimal as a fraction in simplest form. (Lesson 2-1)

54. 0.35  
55. 2.6  
56. \(-7.18\)  
57. 0.125

Write each number in scientific notation. (Lesson 4-4)

58. 1,970,000,000  
59. 2,500,000

60. 31,400,000,000  
61. 5,680,000,000,000,000
Learn to estimate square roots to a given number of decimal places and solve problems using square roots.

A couple wants to install a square stained-glass window. The window has an area of 500 square inches with wood trim around it. You can calculate the length of the trim using your knowledge of squares and square roots.

**Example 1**

**Estimating Square Roots of Numbers**

Each square root is between two integers. Name the integers. Explain your answer.

- **A** \(\sqrt{30}\) 
  - Think: What are perfect squares close to 30?
  - \(5^2 = 25\) \(\quad 25 < 30\)
  - \(6^2 = 36\) \(\quad 36 > 30\)
  - \(\sqrt{30}\) is between 5 and 6 because 30 is between 25 and 36.

- **B** \(-\sqrt{150}\) 
  - Think: What are perfect squares close to 150?
  - \((-12)^2 = 144\) \(\quad 144 < 150\)
  - \((-13)^2 = 169\) \(\quad 169 > 150\)
  - \(-\sqrt{150}\) is between \(-12\) and \(-13\) because 150 is between 144 and 169.

**Example 2**

**Problem Solving Application**

A couple wants to install a square stained-glass window that has an area of 500 square inches. Calculate the length of each side and the length of trim needed to the nearest tenth of an inch.

1. **Understand the Problem**
   - First find the length of a side. Then you can use the length of a side to find the *perimeter*, the length of the trim around the window.

2. **Make a Plan**
   - The length of a side, in inches, is the number that you multiply by itself to get 500. Find this number to the nearest tenth.
   - Use guess and check to find \(\sqrt{500}\).
3 Solve

Because 500 is between $22^2$ (484) and $23^2$ (529), the square root of 500 is between 22 and 23.

The square root is between 22.3 and 22.4. To round to the nearest tenth, consider 22.35.

The square root must be greater than 22.35, so you can round up. To the nearest tenth, $\sqrt{500}$ is about 22.4.

Now estimate the length around the window. The length of a side of the window to the nearest tenth of an inch is 22.4 inches.

$$4 \cdot 22.4 = 89.6$$  
Perimeter = 4 \cdot side

The trim is about 89.6 inches long.

4 Look Back

The length 90 inches divided by 4 is 22.5 inches. A 22.5-inch square has an area of 506.25 square inches, which is close to 500, so the answers are reasonable.

Using a Calculator to Estimate the Value of a Square Root

Use a calculator to find $\sqrt{700}$. Round to the nearest tenth. Using a calculator, $\sqrt{700} = 26.45751311 \ldots$. Rounded, $\sqrt{700}$ is 26.5.

Think and Discuss

1. Discuss whether 9.5 is a good first guess for $\sqrt{75}$.
2. Determine which square root or roots would have 7.5 as a good first guess.
4-6 Exercises

GUIDED PRACTICE

See Example 1 Each square root is between two integers. Name the integers. Explain your answer.

1. \(\sqrt{40}\)  
2. \(-\sqrt{90}\)  
3. \(\sqrt{156}\)  
4. \(-\sqrt{306}\)  
5. \(\sqrt{250}\)

See Example 2 6. A square photo is placed behind a piece of glass that has an area of 20 square inches. To the nearest hundredth, what length of frame is needed to go around all edges of the glass?

See Example 3 Use a calculator to find each value. Round to the nearest tenth.

7. \(\sqrt{74}\)  
8. \(\sqrt{34.1}\)  
9. \(\sqrt{3600}\)  
10. \(\sqrt{190}\)  
11. \(\sqrt{5120}\)

INDEPENDENT PRACTICE

See Example 1 Each square root is between two integers. Name the integers. Explain your answer.

12. \(-\sqrt{52}\)  
13. \(\sqrt{3}\)  
14. \(\sqrt{600}\)  
15. \(-\sqrt{2000}\)  
16. \(\sqrt{410}\)

See Example 2 17. Each square on Laura’s chessboard is 13 square centimeters. A chessboard has 8 squares on each side. To the nearest hundredth, what is the width of Laura’s chessboard?

See Example 3 Use a calculator to find each value. Round to the nearest tenth.

18. \(\sqrt{58}\)  
19. \(\sqrt{91.5}\)  
20. \(\sqrt{550}\)  
21. \(\sqrt{150}\)  
22. \(\sqrt{330}\)

PRACTICE AND PROBLEM SOLVING

Write the letter that identifies the position of each square root.

23. \(-\sqrt{3}\)  
24. \(\sqrt{5}\)  
25. \(\sqrt{7}\)  
26. \(-\sqrt{8}\)  
27. \(\sqrt{14}\)  
28. \(\sqrt{0.75}\)

Find each product to the nearest hundredth.

29. \(\sqrt{51} \cdot \sqrt{25}\)  
30. \(-\sqrt{70} \cdot \sqrt{16}\)  
31. \(\sqrt{215} \cdot (-\sqrt{1})\)  
32. \(-\sqrt{113} \cdot \sqrt{9}\)  
33. \(\sqrt{22} \cdot (-\sqrt{49})\)  
34. \(\sqrt{210} \cdot \sqrt{169}\)

35. **Multi-Step** On a baseball field, the infield area created by the baselines is a square. In a youth baseball league for 9- to 12-year-olds, this area is 3600 ft². The distance between each base in a league for 4-year-olds is 20 ft less than it is for 9- to 12-year-olds. What is the distance between each base for 4-year-olds?
Tsunamis, sometimes called tidal waves, move across deep oceans at high speeds with barely a ripple on the water surface. It is only when tsunamis hit shallow water that their energy moves them upward into a mammoth destructive force.

36. The rate of speed of a tsunami, in feet per second, can be found by the formula \( r = \sqrt{32d} \), where \( d \) is the water depth in feet. Suppose the water depth is 20,000 ft. How fast is the tsunami moving?

37. The speed of a tsunami in miles per hour can be found using \( r = \sqrt{14.88d} \), where \( d \) is the water depth in feet. Suppose the water depth is 25,000 ft.
   a. How fast is the tsunami moving in miles per hour?
   b. How long would it take a tsunami to travel 3000 miles if the water depth were a consistent 10,000 ft?

38. **What’s the Error?** Ashley found the speed of a tsunami, in feet per second, by taking the square root of 32 and multiplying by the depth, in feet. What was her error?

39. **Challenge** Find the depth of the water if a tsunami’s speed is 400 miles per hour.

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**Test Prep and Spiral Review**

40. **Multiple Choice** Which expression has a value between 15 and 14?
   
   \( \text{A) } \sqrt{188} \quad \text{B) } \sqrt{200} \quad \text{C) } \sqrt{227} \quad \text{D) } \sqrt{324} \)

41. **Gridded Response** Find the product \( \sqrt{42} \cdot \sqrt{94} \) to the nearest hundredth.

Evaluate each expression for the given values of the variables. **(Lesson 1-1)**

42. \( 4x + 5y \) for \( x = 3 \) and \( y = 9 \)  
43. \( 7m - 2n \) for \( m = 5 \) and \( n = 7 \)

44. \( 8h + 9j \) for \( h = 11 \) and \( j = 2 \)  
45. \( 6s - 2t \) for \( s = 7 \) and \( t = 12 \)

Find the two square roots of each number. **(Lesson 4-5)**

46. 100  
47. 64  
48. 484  
49. 1296
A graphing calculator can be used to evaluate expressions that have negative exponents and square roots.

**Activity**

1. Use the **STO** button to evaluate $x^{-3}$ for $x = 2$. View the answer as a decimal and as a fraction.

   ![Graphing Calculator Screen]

   Notice that $2^{-3} = 0.125$, which is equivalent to $\frac{1}{2^3}$ or $\frac{1}{8}$.

2. Use the **TABLE** feature to evaluate $-\sqrt{x}$ for several $x$-values. Match the settings shown.

   ![Graphing Calculator Screen]

   The **Y1** list shows the value of $-\sqrt{x}$ for several $x$-values.

**Think and Discuss**

1. When you evaluated $2^{-3}$ in Activity 1, the result was not a negative number. Is this surprising? Why or why not?

**Try This**

Evaluate each expression for the given $x$-value(s). Give your answers as fractions and as decimals rounded to the nearest hundredth.

1. $4^{-x}; x = 2$
2. $\sqrt{x}; x = 1, 2, 3, 4$
3. $x^{-2}; x = 1, 2, 5$
Biologists classify animals based on shared characteristics. The horned lizard is an animal, a reptile, a lizard, and a gecko.

You already know that some numbers can also be classified as whole numbers, integers, or rational numbers. The number 2 is a whole number, an integer, and a rational number. It is also a real number.

Rational numbers can be written as fractions and as decimals that either terminate or repeat.

\[
\frac{3}{5} = 0.6 \quad \frac{2}{3} = 0.6 \quad \sqrt{1.44} = 1.2
\]

Irrational numbers can only be written as decimals that do not terminate or repeat. If a whole number is not a perfect square, then its square root is an irrational number.

\[
\sqrt{2} \approx 1.414213562370950488016 \ldots
\]

The set of real numbers consists of the set of rational numbers and the set of irrational numbers.

### Example 1: Classifying Real Numbers

Write all names that apply to each number.

**A.** \(\sqrt{3}\)
- irrational, real

**B.** \(-52.28\)
- rational, real

**C.** \(\frac{\sqrt{16}}{4}\)
- whole, integer, rational, real

A repeating decimal may not appear to repeat on a calculator because calculators show a finite number of digits.

Horned lizards are commonly called “horny toads” because of their flattened, toad-like bodies.
The square root of a negative number is not a real number. A fraction with a denominator of 0 is undefined because you cannot divide by zero. So it is not a number at all.

**Example 2**

**Determining the Classification of All Numbers**

State if each number is rational, irrational, or not a real number.

A. \( \sqrt{15} \)
   - 15 is a whole number that is not a perfect square.
   - irrational

B. \( \frac{3}{0} \)
   - undefined, so not a real number

C. \( \sqrt{\frac{1}{9}} \)  \( \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) = \frac{1}{9} \)
   - rational

D. \( \sqrt{-13} \)
   - not a real number

The **Density Property** of real numbers states that between any two real numbers is another real number. This property is not true for whole numbers or integers. For instance, there is no integer between \(-2\) and \(-3\).

**Example 3**

**Applying the Density Property of Real Numbers**

Find a real number between \(1\frac{1}{3}\) and \(1\frac{2}{3}\).

There are many solutions. One solution is halfway between the two numbers. To find it, add the numbers and divide by 2.

\[
\left(1\frac{1}{3} + 1\frac{2}{3}\right) \div 2 = \frac{2\frac{3}{3}}{2} = \frac{3}{2} = 1\frac{1}{2}
\]

A real number between \(1\frac{1}{3}\) and \(1\frac{2}{3}\) is \(1\frac{1}{2}\).

**Think and Discuss**

1. **Explain** how rational numbers are related to integers.
2. **Tell** if a number can be irrational and whole. Explain.
3. **Use** the Density Property to explain why there are infinitely many real numbers between 0 and 1.
4-7 Exercises

**GUIDED PRACTICE**

**See Example 1**
Write all names that apply to each number.

1. \( \sqrt{10} \)
2. \( \sqrt{49} \)
3. 0.25
4. \(-\sqrt{16}/3\)

**See Example 2**
State if each number is rational, irrational, or not a real number.

5. \( \sqrt{9} \)
6. \( \sqrt{9/16} \)
7. \( \sqrt{72} \)
8. \(-\sqrt{3} \)
9. \(-\sqrt{25} \)
10. \(\sqrt{-9} \)
11. \(\sqrt{25/-36} \)
12. \(0/0 \)

**See Example 3**
Find a real number between each pair of numbers.

13. \(3\frac{1}{8}\) and \(3\frac{2}{8}\)
14. 4.14 and \(29/7\)
15. \(1/8\) and \(1/4\)

**INDEPENDENT PRACTICE**

**See Example 1**
Write all names that apply to each number.

16. \(\sqrt{35} \)
17. \(5/8 \)
18. 3
19. \(-81/-3 \)

**See Example 2**
State if each number is rational, irrational, or not a real number.

20. \(-\sqrt{16}/-4 \)
21. \(-\sqrt{0}/4 \)
22. \(-\sqrt{8/-2} \)
23. \(-\sqrt{3} \)
24. \(\sqrt{25}/8 \)
25. \(\sqrt{14} \)
26. \(\sqrt{1/-4} \)
27. \(-\sqrt{4}/0 \)

**See Example 3**
Find a real number between each pair of numbers.

28. \(3\frac{2}{5}\) and \(3\frac{3}{5} \)
29. \(-1/10\) and 0
30. 4 and \(\sqrt{9} \)

**PRACTICE AND PROBLEM SOLVING**

Write all names that apply to each number.

31. 6
32. \(-\sqrt{36} \)
33. \(\sqrt{10} \)
34. \(1/3 \)
35. \(\sqrt{2.56} \)
36. \(\sqrt{36 + 6} \)
37. 0.21
38. \(\sqrt{100/20} \)
39. \(-4.3134 \)
40. \(\sqrt{4.5} \)
41. \(-312 \)
42. \(0/7 \)

43. Explain the difference between \(-\sqrt{16}\) and \(\sqrt{-16} \).

Give an example of each type of number.

44. an irrational number that is less than \(-3\)
45. a rational number that is less than 0.3
46. a real number between \(5/9\) and \(6/9\)
47. a real number between \(-3\frac{2}{7}\) and \(-3\frac{3}{7}\)
48. Find a rational number between $\sqrt{\frac{1}{9}}$ and $\sqrt{1}$.
49. Find a real number between $\sqrt{6}$ and $\sqrt{7}$.
50. Find a real number between $\sqrt{5}$ and $\sqrt{11}$.
51. Find a real number between $\sqrt{50}$ and $\sqrt{55}$.
52. Find a real number between $-\sqrt{20}$ and $-\sqrt{17}$.
53. a. Find a real number between 1 and $\frac{3}{8}$.
   b. Find a real number between 1 and your answer to part a.
   c. Find a real number between 1 and your answer to part b.
54. For what values of $x$ is the value of each expression a real number?
   55. \(3 - \sqrt{x}\)
   56. $\sqrt{x + 2}$
   57. $\sqrt{3x - 6}$
   58. $\sqrt{5x + 2}$
   59. $\sqrt{1 - \frac{x}{5}}$
60. What’s the Error? A student said that all integers are whole numbers. What mistake did the student make? Explain.
61. Write About It Can you ever use a calculator to determine if a number is rational or irrational? Explain.
62. Challenge The circumference of a circle divided by its diameter is an irrational number, represented by the Greek letter $\pi$ (pi). Could a circle with a diameter of 2 have a circumference of 6? Why or why not?
63. Multiple Choice Which value is between $\frac{1}{8}$ and $\frac{1}{10}$?
   64. Multiple Choice Which value is NOT a rational number?
   65. Multiple Choice For which values of $x$ is $\sqrt{x - 19}$ a real number?

Evaluate the function $y = -5x + 2$ for each value of $x$. (Lesson 3-4)
66. $x = 0$ 67. $x = -3$ 68. $x = 7$ 69. $x = -1$
66. Evaluate. (Lesson 4-1)
70. $8^5$ 71. $(-3)^3$ 72. $(-5)^4$ 73. $9^2$
The Pythagorean Theorem states that if \(a\) and \(b\) are the lengths of the legs of a right triangle, then \(c\) is the length of the hypotenuse, where \(a^2 + b^2 = c^2\). Prove the Pythagorean Theorem using the following steps.

**a.** Draw two squares side by side. Label one with side \(a\) and one with side \(b\).

**b.** Draw hypotenuses of length \(c\), so that we have right triangles with sides \(a, b,\) and \(c\). Use a protractor to make sure that the hypotenuses form a right angle.

**c.** Cut out the triangles and the remaining piece.

**d.** Fit the pieces together to make a square with sides \(c\) and area \(c^2\). You have shown that the area \(a^2 + b^2\) can be cut up and rearranged to form the area \(c^2\), so \(a^2 + b^2 = c^2\).

**Think and Discuss**

1. The diagram shows another way of understanding the Pythagorean Theorem. How are the areas of the squares shown in the diagram related?

**Try This**

1. If you know that the lengths of two legs of a right triangle are 8 and 15, can you find the length of the hypotenuse? Show your work.

2. Take a piece of paper and fold the right corner down so that the top edge of the paper matches the side edge. Crease the paper. Without measuring, find the diagonal's length.
Learn to use the Pythagorean Theorem to solve problems.

Vocabulary
- Pythagorean Theorem
- leg
- hypotenuse

Pythagoras was born on the Aegean island of Samos sometime between 580 B.C. and 569 B.C. He is best known for the Pythagorean Theorem, which relates the side lengths of a right triangle.

A Babylonian tablet known as Plimpton 322 provides evidence that the relationship between the side lengths of right triangles was known as early as 1900 B.C. Many people, including U.S. president James Garfield, have written proofs of the Pythagorean Theorem. In 1940, E. S. Loomis presented 370 proofs of the theorem in *The Pythagorean Proposition*.

### THE PYTHAGOREAN THEOREM

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>In any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.</td>
<td>$6^2 + 8^2 = 10^2$ $36 + 64 = 100$</td>
<td>$a^2 + b^2 = c^2$</td>
</tr>
</tbody>
</table>

**Example 1**

Finding the Length of a Hypotenuse

Find the length of each hypotenuse to the nearest hundredth.

**Helpful Hint**

Since length can only be positive, use only the principal square root.

![Diagram of a right triangle with sides 2, 2, and c, and hypotenuse c, illustrating the Pythagorean Theorem: $2^2 + 2^2 = c^2$, $4 + 4 = c^2$, $8 = c^2$, $\sqrt{8} = \sqrt{c^2}$, $2.83 \approx c$.]

**Pythagorean Theorem**
**Substitute 2 for a and 2 for b.**
**Simplify powers.**
**Add.**
**Find the square roots.**
**Round to the nearest hundredth.**
Find the length of each hypotenuse to the nearest hundredth.

The points form a right triangle with 

\[ a = 4 \text{ and } b = 3. \]

\[ a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem} \]

\[ 4^2 + 3^2 = c^2 \quad \text{Substitute for } a \text{ and } b. \]

\[ 16 + 9 = c^2 \quad \text{Simplify powers.} \]

\[ 25 = c^2 \quad \text{Add.} \]

\[ 5 = c \quad \text{Find the square roots.} \]

**EXAMPLE 2**

**Finding the Length of a Leg in a Right Triangle**

Solve for the unknown side in the right triangle to the nearest tenth.

\[ a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem} \]

\[ 9^2 + b^2 = 15^2 \quad \text{Substitute for } a \text{ and } b. \]

\[ 81 + b^2 = 225 \quad \text{Simplify powers.} \]

\[ b^2 = 144 \quad \text{Subtract 81 from each side.} \]

\[ b = 12 \quad \text{Find the square roots.} \]

**EXAMPLE 3**

**Using the Pythagorean Theorem for Measurement**

Mark and Sarah start walking at the same point, but Mark walks 50 feet north while Sarah walks 75 feet east. How far apart are Mark and Sarah when they stop?

Mark and Sarah's distance from each other when they stop walking is equal to the hypotenuse of a right triangle.

\[ a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem} \]

\[ 50^2 + 75^2 = c^2 \quad \text{Substitute for } a \text{ and } b. \]

\[ 2500 + 5625 = c^2 \quad \text{Simplify powers.} \]

\[ 8125 = c^2 \quad \text{Add.} \]

\[ 90.1 \approx c \quad \text{Find the square roots.} \]

Mark and Sarah are approximately 90.1 feet apart.

**Think and Discuss**

1. Tell which side of a right triangle is always the longest side.
2. Explain if 2, 3, and 4 cm could be side lengths of a right triangle.
4-8 Exercises

GUIDED PRACTICE

Find the length of each hypotenuse to the nearest hundredth.

1. \(a = 12, \ b = 9, \ c = \) 
2. \(a = 8, \ b = 7, \ c = \) 
3. \(a = 6, \ b = 6, \ c = \) 
4. triangle with coordinates \((-4, 0), (-4, 5), (0, 5)\)

Solve for the unknown side in each right triangle to the nearest tenth.

5. \(a = 17, \ b = 15, \ c = \) 
6. \(a = 6, \ b = 8, \ c = \) 
7. \(a = 20, \ b = 12, \ c = \) 
8. A traffic helicopter flies 10 miles due north and then 24 miles due east. Then the helicopter flies in a straight line back to its starting point. What was the distance of the helicopter’s last leg back to its starting point?

INDEPENDENT PRACTICE

Find the length of each hypotenuse to the nearest hundredth.

9. \(a = 5, \ b = 2, \ c = \) 
10. \(a = 12, \ b = 5, \ c = \) 
11. \(a = 7, \ b = 26, \ c = \) 
12. triangle with coordinates \((-5, 3), (5, -3), (-5, -3)\)

Solve for the unknown side in each right triangle to the nearest tenth.

13. \(a = 5, \ b = 13, \ c = \) 
14. \(a = 4, \ b = 13, \ c = \) 
15. \(a = 11, \ b = 15, \ c = \) 
16. Mr. and Mrs. Flores commute to work each morning. Mr. Flores drives 8 miles east to his office. Mrs. Flores drives 15 miles south to her office. How many miles away do Mr. and Mrs. Flores work from each other?

PRACTICE AND PROBLEM SOLVING

Find the missing length for each right triangle to the nearest tenth.

17. \(a = 4, b = 7, c = \) 
18. \(a = \, b = 40, c = 41\) 
19. \(a = 30, b = 72, c = \) 
20. \(a = 16, b = \, c = 38\) 
21. \(a = \, b = 47, c = 60\) 
22. \(a = 65, b = \, c = 97\)
Any three nonzero whole numbers that make the equation \(a^2 + b^2 = c^2\) true are the side lengths of a right triangle. These numbers are called Pythagorean triples. Determine whether each set is a Pythagorean triple.

23. 3, 6, 9  
24. 3, 4, 5  
25. 5, 12, 13  
26. 7, 24, 25  
27. 10, 24, 26  
28. 8, 14, 16  
29. 10, 16, 19  
30. 9, 40, 41

31. For safety reasons, the base of a 24-foot ladder must be placed at least 8 feet from the wall. To the nearest tenth of a foot, how high can a 24-foot ladder safely reach?

32. How far is the sailboat from the lighthouse, to the nearest kilometer?

33. Critical Thinking A construction company is pouring a rectangular concrete foundation. The dimensions of the foundation are 24 ft by 48 ft. Describe a procedure to confirm that the sides of the foundation meet at a right angle.

34. Write a Problem Use a street map to write and solve a problem that requires the use of the Pythagorean Theorem.

35. Write About It Explain how to use the side measures to show that a triangle is a right triangle.

36. Challenge A right triangle has legs of length 3x m and 4x m and hypotenuse of length 75 m. Find the lengths of the legs of the triangle.

Test Prep and Spiral Review

37. Multiple Choice A flagpole is 40 feet tall. A rope is tied to the top of the flagpole and secured to the ground 9 feet from the base of the flagpole. What is the length of the rope to the nearest foot?

- (A) 19 feet  
- (B) 39 feet  
- (C) 41 feet  
- (D) 1519 feet

38. Gridded Response Brad leans his 15-foot ladder against his house. The base of the ladder is placed 4 feet from the base of the house. How far up the house does the ladder reach? Round your answer to the nearest hundredth.

Find the next term in each sequence. (Lesson 3-6)

39. \(-3, 0, 3, 6, \ldots\)  
40. \(0.55, 0.65, 0.75, 0.85, \ldots\)  
41. \(9, 16, 23, 30, 37, 44, \ldots\)  
42. \(1, 1.5, 2, 2.5, \ldots\)  
43. \(-1, 1, 3, 5, \ldots\)  
44. \(0, -2, -4, -6, \ldots\)

Estimate each square root to two decimal places. (Lesson 4-6)

45. \(\sqrt{30}\)  
46. \(\sqrt{42}\)  
47. \(\sqrt{55}\)  
48. \(\sqrt{67}\)
Quiz for Lessons 4-5 Through 4-8

4-5 Squares and Square Roots
Find the two square roots of each number.
1. 16  
2. 9801  
3. 10,000  
4. 529  
5. The Merryweathers want a new square rug for their living room. If the living room is 20 ft × 16 ft, will a square rug with an area of 289 square feet fit? Explain your answer.
6. How many 2 in. × 2 in. square tiles will fit along the edge of a square mosaic that has an area of 196 square inches?

4-6 Estimating Square Roots
Each square root is between two integers. Name the integers. Explain your answer.
7. $\sqrt{72}$  
8. $\sqrt{200}$  
9. $\sqrt{340}$  
10. $\sqrt{610}$  
11. A square table has a top with an area of 11 square feet. To the nearest hundredth, what length of edging is needed to go around all edges of the tabletop?
12. The area of a chess board is 110 square inches. Find the length of one side of the board to the nearest hundredth.

4-7 The Real Numbers
Write all names that apply to each number.
13. $\sqrt{12}$  
14. 0.15  
15. $\sqrt{1600}$  
16. $\sqrt{-144}$  
17. Give an example of an irrational number that is less than $-5$.
18. Find a real number between 5 and $\sqrt{36}$.

4-8 The Pythagorean Theorem
Find the missing length for each right triangle. Round your answer to the nearest tenth.
19. $a = \square$, $b = 6$, $c =$  
20. $a = \square$, $b = 24$, $c =$  
21. $a = 20$, $b =$, $c = 46$  
22. $a =$, $b = 53$, $c =$  
23. $a = 14$, $b = 15$, $c =$  
24. $a = 8$, $b =$, $c =$  
25. A construction company is pouring a concrete foundation. The measures of two sides that meet in a corner are 33 ft and 56 ft. For the corner to be a right angle, what would the length of the diagonal have to be?
Divide and Conquer

A biologist is growing colonies of two bacteria. As shown in the table, the cells of bacterium A divide in two every hour. The cells of bacterium B divide in two every two hours.

<table>
<thead>
<tr>
<th>Elapsed Time</th>
<th>Number of Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bacterium A</td>
</tr>
<tr>
<td>Start</td>
<td>1</td>
</tr>
<tr>
<td>1 hour</td>
<td>$2^1$</td>
</tr>
<tr>
<td>2 hours</td>
<td>$2^2$</td>
</tr>
<tr>
<td>3 hours</td>
<td>$2^3$</td>
</tr>
<tr>
<td>4 hours</td>
<td>$2^4$</td>
</tr>
</tbody>
</table>

1. After 8 hours, how many more cells are there of bacterium A than of bacterium B?

2. How many hours does it take until there are more than 1000 cells of bacterium A?

3. After 24 hours, how many times as many cells are there of bacterium A as bacterium B?

4. At the end of 24 hours, there are about $1.68 \times 10^7$ cells of bacterium A. The biologist divides this colony into 3 roughly equal portions. About how many cells are in each portion?

5. As a rule of thumb, if an experiment yields $n$ colonies of bacteria, future experiments are likely to yield between $n - \sqrt{n}$ and $n + \sqrt{n}$ colonies. Suppose an experiment produces 170 colonies of bacterium A. Explain how you can estimate the range of the number of colonies that future experiments will produce.
Magic Squares

A magic square is a square with numbers arranged so that the sums of the numbers in each row, column, and diagonal are the same.

According to an ancient Chinese legend, a tortoise from the Lo river had the pattern of this magic square on its shell.

1. Complete each magic square below.

\[
\begin{array}{ccc}
\sqrt{36} & & 2^2 \\
8^0 & \sqrt{9} & \\
3^2 - 2 & & \\
\end{array}
\]

\[
\begin{array}{ccc}
\ & -\left(\sqrt{4} + 4\right) & -9^0 \\
\ & -\left(\sqrt{16}\right) & 0^3 \\
\ & -\left(\sqrt{9}\right) & 2^0 + 1 \\
\end{array}
\]

2. Use the numbers \(-4, -3, -2, -1, 0, 1, 2, 3,\) and \(4\) to make a magic square with row, column, and diagonal sums of 0.

Equation Bingo

Each bingo card has numbers on it. The caller has a collection of equations. The caller reads an equation, and then the players solve the equation for the variable. If players have the solution on their cards, they place a chip on it. The winner is the first player with a row of chips either down, across, or diagonally.

A complete copy of the rules and game boards are available online.
**Materials**
- strip of white paper (18 in. by 7 in.)
- piece of decorative paper (6 in. by 6 in.)
- tape
- scraps of decorative paper
- markers
- glue

**PROJECT** It’s a Wrap

Design your own energy-bar wrapper to hold your notes on exponents and roots.

**Directions**

1. Make accordion folds on the strip of white paper so that there are six panels, each about 3 in. wide. **Figure A**

2. Fold up the accordion strip.

3. Wrap the decorative paper around the accordion strip. The accordion strip will stick out on either side. Tape the ends of the decorative paper together to make a wrapper. **Figure B**

4. Write the number and title of the chapter on scraps of decorative paper, and glue these to the wrapper.

**Taking Note of the Math**

Use the panels of the accordion strip to take notes on the key concepts in this chapter. Include examples that will help you remember facts about exponents, roots, and the Pythagorean Theorem. Fold up the strip and slide it back into the wrapper.
Vocabulary

base ........................................ 162
Density Property ....................... 192
exponent ................................... 162
exponential form ....................... 162
hypotenuse ................................ 196
irrational number ..................... 191
leg ............................................ 196

perfect square ......................... 182
power ..................................... 162
principal square root .................. 182
Pythagorean Theorem .................. 196
real number ............................. 191
scientific notation .................... 174

Complete the sentences below with vocabulary words from the list above.

1. A power consists of a(n) ___?___ raised to a(n) ___?___.
2. A(n) ___?___ is a number that cannot be written as a fraction.
3. ___?___ is a short-hand way of writing extremely large or extremely small numbers.
4. The ___?___ states that the sum of the squares of the ___?___ of a right triangle is equal to the square of the ___?___.
5. The set of ___?___ is the set of all rational and irrational numbers.

4-1 Exponents (pp. 162–165)

**Example**

- **Write in exponential form.**
  
  \[ 4 \cdot 4 \cdot 4 \]
  
  \[ 4^3 \]

  *Identify how many times 4 is used as a factor.*

- **Evaluate.**
  
  \[ (-2)^3 \]
  
  \[ (-2) \cdot (-2) \cdot (-2) \]
  
  \[ -8 \]

  *Find the product of three \(-2\)'s.*

**Exercises**

Write in exponential form.

6. \[ 7 \cdot 7 \cdot 7 \]
7. \[ (-3) \cdot (-3) \]
8. \[ k \cdot k \cdot k \]
9. \[ -9 \]
10. \[ (-2) \cdot (-2) \cdot d \cdot d \]
11. \[ 3n \cdot 3n \cdot 3n \]
12. \[ 6 \cdot x \cdot x \]
13. \[ 10,000 \]

Evaluate.

14. \[ 5^4 \]
15. \[ (-2)^5 \]
16. \[ (-1)^9 \]
17. \[ 2^8 \]
18. \[ (-3)^1 \]
19. \[ 4^3 \]
20. \[ (-3)^3 \]
21. \[ (-5)^2 \]
22. \[ 15^1 \]
23. \[ 6^4 \]
24. \[ 10^5 \]
25. \[ (-2)^7 \]
4-2 Look for a Pattern in Integer Exponents (pp. 166–169)

**Example**

Evaluate.

1. \((-3)^{-2}\)
   
   \[
   \frac{1}{(-3)^2} = \frac{1}{9}
   \]

2. \(2^0\)
   
   \[
   1
   \]

**Exercises**

Evaluate.

26. \(5^{-3}\)
27. \((-4)^{-3}\)
28. \(11^{-1}\)
29. \(10^{-4}\)
30. \(100^0\)
31. \(-6^{-2}\)
32. \(-3^{-4}\)
33. \((-10)^{-2}\)
34. \((9 - 7)^{-3}\)
35. \((6 - 9)^{-3}\)
36. \((8 - 5)^{-1}\)
37. \((7 - 10)^0\)
38. \(4^{-1} + (5 - 7)^{-2}\)
39. \(3^{-2} \cdot 2^{-3} \cdot 9^0\)
40. \(10 - 9(3^{-2} + 6^0)\)

4-3 Properties of Exponents (pp. 170–173)

**Example**

Write the product or quotient as one power.

1. \(2^5 \cdot 2^3\)
   
   \[
   2^{5 + 3} = 2^8
   \]

2. \(\frac{10^9}{10^2}\)
   
   \[
   10^{9 - 2} = 10^7
   \]

**Exercises**

Write the product or quotient as one power.

41. \(4^2 \cdot 4^5\)
42. \(9^2 \cdot 9^4\)
43. \(p \cdot p^3\)
44. \(15 \cdot 15^2\)
45. \(6^2 \cdot 3^2\)
46. \(x^4 \cdot x^6\)
47. \(\frac{8^3}{8^2}\)
48. \(\frac{9^3}{9}\)
49. \(\frac{m^7}{m^2}\)
50. \(\frac{3^5}{3^{-2}}\)
51. \(\frac{4^{-5}}{4^{-5}}\)
52. \(\frac{y^6}{y^{-3}}\)
53. \(5^0 \cdot 5^3\)
54. \(y^6 + y\)
55. \(k^4 + k^4\)

4-4 Scientific Notation (pp. 174–178)

**Example**

Write in standard notation.

1. \(3.58 \times 10^4\)
   
   \[
   3.58 \times 10,000 = 35,800
   \]

2. \(3.58 \times 10^{-4}\)
   
   \[
   3.58 \times \frac{1}{10,000} = \frac{3.58}{10,000} = 0.000358
   \]

Write in scientific notation.

1. \(0.000007 = 7 \times 10^{-6}\)
2. \(62,500 = 6.25 \times 10^4\)

**Exercises**

Write in standard notation.

56. \(1.62 \times 10^3\)
57. \(1.62 \times 10^{-3}\)
58. \(9.1 \times 10^5\)
59. \(9.1 \times 10^{-5}\)

Write in scientific notation.

60. \(0.000000008\)
61. \(73,000,000\)
62. \(0.0000096\)
63. \(56,400,000,000\)
4-5 Squares and Square Roots (pp. 182–185)

**Example**
- Find the two square roots of 400.
  
  \[ 20 \cdot 20 = 400 \]
  \[ (-20) \cdot (-20) = 400 \]
  
  The square roots are 20 and \(-20\).

**Exercises**
- Find the two square roots of each number.
  64. 16  
  65. 900  
  66. 676

- Evaluate each expression.
  67. \( \sqrt{4 + 21} \)  
  68. \( \frac{\sqrt{100}}{20} \)  
  69. \( \sqrt{3^4} \)

4-6 Estimating Square Roots (pp. 186–189)

**Example**
- Find the side length of a square with area 359 ft\(^2\) to one decimal place. Then find the distance around the square to the nearest tenth.

  - Side = \( \sqrt{359} \approx 18.9 \)
  - Distance around = \( 4(18.9) \approx 75.6 \text{ feet} \)

**Exercises**
- Find the distance around each square with the area given. Round to the nearest tenth.
  70. Area of square \( ABCD \) is 500 in\(^2\).
  71. Area of square \( MNOP \) is 1750 cm\(^2\).
  72. Name the integers \( \sqrt{82} \) is between.

4-7 The Real Numbers (pp. 191–194)

**Example**
- State if the number is rational, irrational, or not a real number.

  - \( -\sqrt{2} \) Irrational  
    - The decimal equivalent does not repeat or end.
  - \( \sqrt{-4} \) Not real  
    - Square roots of negative numbers are not real.

**Exercises**
- State if the number is rational, irrational, or not a real number.
  73. \( \sqrt{81} \)  
  74. \( \sqrt{122} \)  
  75. \( \sqrt{-16} \)  
  76. \( -\sqrt{5} \)  
  77. \( \frac{0}{-4} \)  
  78. \( \frac{7}{0} \)  
  79. Find a real number between \( \sqrt{9} \) and \( \sqrt{16} \).

4-8 The Pythagorean Theorem (pp. 196–199)

**Example**
- Find the length of side \( b \) in the right triangle where \( a = 8 \) and \( c = 17 \).

  \[ a^2 + b^2 = c^2 \]
  \[ 8^2 + b^2 = 17^2 \]
  \[ 64 + b^2 = 289 \]
  \[ b^2 = 225 \]
  \[ b = \sqrt{225} = 15 \]

**Exercises**
- Solve for the unknown side in each right triangle.
  80. If \( a = 6 \) and \( b = 8 \), find \( c \).
  81. If \( b = 24 \) and \( c = 26 \), find \( a \).
  82. Find the length between opposite corners of a square with side lengths 10 inches to the nearest tenth.
Evaluate.
1. $10^9$  
2. $11^{-3}$  
3. $2^7$  
4. $3^{-4}$

Evaluate each expression. Write your answer as one power.
5. $\frac{33}{36}$  
6. $7^9 \cdot 7^2$  
7. $(5^{10})^6$  
8. $\frac{11^{-7}}{11^7}$  
9. $27^3 \cdot 27^{-18}$  
10. $(52^{-7})^{-3}$  
11. $13^0 \cdot 13^9$  
12. $\frac{g^{12}}{8^7}$

Write each number in standard notation.
13. $2.7 \times 10^{12}$  
14. $3.53 \times 10^{-2}$  
15. $4.257 \times 10^5$  
16. $9.87 \times 10^{10}$  
17. $4.8 \times 10^8$  
18. $6.09 \times 10^{-3}$  
19. $8.1 \times 10^6$  
20. $3.5 \times 10^{-4}$

Write each number in scientific notation.
21. $19,000,000,000$  
22. $0.0000039$  
23. $1,980,000,000$  
24. $0.00045$  

Find the two square roots of each number.
26. 196  
27. 1  
28. 10,000  
29. 625  
30. The minimum area of a square, high school wrestling mat is 1444 square feet. What is the length of the mat?

Each square root is between two integers. Name the integers. Explain your answer.
31. $\sqrt{230}$  
32. $\sqrt{125}$  
33. $\sqrt{89}$  
34. $-\sqrt{60}$  
35. $-\sqrt{3}$  
36. $\sqrt{175}$  
37. $-\sqrt{410}$  
38. $\sqrt{325}$  
39. A square has an area of 13 ft$^2$. To the nearest tenth, what is its perimeter?

Write all names that apply to each number.
40. $-\sqrt{121}$  
41. $-1.7$  
42. $\sqrt{-9}$  
43. $\frac{\sqrt{225}}{3}$

Find the missing length for each right triangle.
44. $a = 10$, $b = 24$, $c = \square$  
45. $a = \square$, $b = 15$, $c = 17$  
46. $a = 12$, $b = \square$, $c = 20$  
47. Lupe wants to use a fence to divide her square garden in half diagonally. If each side of the garden is 16 ft long, how long will the fence have to be? Round your answer to the nearest hundredth of a foot.
48. A right triangle has a hypotenuse that is 123 in. long. If one of the legs is 75 in. long, how long is the other leg? Round your answer to the nearest tenth of an inch.
Cumulative Assessment, Chapters 1–4

Multiple Choice

1. Which expression is NOT equivalent to \(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3\)?
   A) \(3^6\)  
   B) \(9^3\)  
   C) 18  
   D) 729

2. A number to the 8th power divided by the same number to the 4th power is 16. What is the number?
   A) 2  
   B) 4  
   C) 8  
   D) 6

3. Which expression is equivalent to 81?
   A) \(2^9\)  
   B) \(3^{-4}\)  
   C) \(\left(\frac{1}{3}\right)^{-4}\)  
   D) \(\left(\frac{1}{3}\right)^{14}\)

4. The airports in the United States serve more than 635,000,000 people each year. Which of the following is the same number written in scientific notation?
   A) \(6.35 \times 10^6\)  
   B) \(6.35 \times 10^8\)  
   C) \(6.35 \times 10^{-8}\)  
   D) \(6.35 \times 10^9\)

5. For which equation is the ordered pair \((-3, 4)\) a solution?
   A) \(2x - y = -6\)  
   B) \(x - 2y = 5\)  
   C) \(\frac{1}{2}x - y = 6\)  
   D) \(x - \frac{1}{2}y = -5\)

6. The population of India is close to \(1.08 \times 10^9\). Which of the following represents this population written in standard notation?
   A) 1,080,000,000  
   B) 180,000,000  
   C) 1,080,000  
   D) 108,000

7. Jenny finds that a baby lizard grows about 0.5 inch every week. Which equation best represents the number of weeks it will take for the lizard to grow to 1 foot long if it was 4 inches long when it hatched?
   A) \(0.5w + 4 = 1\)  
   B) \(0.5w + 4 = 12\)  
   C) \(\frac{w + 4}{12} = 0.5\)  
   D) \(\frac{w}{0.5 + 4} = 1\)

8. A number \(k\) is decreased by 8, and the result is multiplied by 8. This product is then divided by 2. What is the final result?
   A) \(8k - 4\)  
   B) \(4k - 8\)  
   C) \(4k - 32\)  
   D) \(8k - 64\)

9. Which ordered pair lies on the x-axis?
   A) \((-1, 2)\)  
   B) \((1, -2)\)  
   C) \((0, 2)\)  
   D) \((-1, 0)\)

10. A quilt is made with 10 square pieces of fabric. If the area of each square piece is 169 square inches, what is the length of each square piece?
   A) 12 inches  
   B) 13 inches  
   C) 14 inches  
   D) 15 inches

11. Which number is NOT between 1.5 and 1.75?
   A) \(1\frac{1}{4}\)  
   B) 1.73  
   C) 1.62  
   D) \(1\frac{13}{25}\)

12. The \(\sqrt{18}\) is between which pair of numbers?
   A) 8 and 9  
   B) 7 and 8  
   C) 4 and 5  
   D) 3 and 4
13. Mrs. Graham ordered five pizzas for her top-performing class. The students ate $\frac{7}{8}$ of the pepperoni pizza, $\frac{3}{4}$ of the cheese pizza, $\frac{2}{3}$ of the veggie pizza, $\frac{2}{3}$ of the Hawaiian pizza, and $\frac{1}{2}$ of the barbecue chicken pizza. How much total pizza was left over?

**Gridded Response**

14. What exponent makes the statement $3^? = 27^2$ true?

15. Determine the value of $x$ when $y = 3$ in the graph.

16. Chrissy is 25 years older than her dog. The sum of their ages is 37. How old is Chrissy’s dog?

17. Evaluate the expression, $\frac{4}{5} - \left|\frac{1}{2} - x\right|$ for $x = \frac{1}{5}$.

18. The area of a square is 169 square feet. What is the length in feet of a side?

19. From her house, Lea rode her bike 8 miles north and then 15 miles west to a friend’s house. How far in miles was she from her house along a straight path?

**Short Response**

20. A bag of pinto beans weighs 210 pounds.
   a. How much does 10,000 bags of pinto beans weigh? Write your answer in standard form.
   b. Write the numbers 210 and 10,000 in scientific notation.
   c. Explain how to use rules of exponents to write the weight of 10,000 bags of pinto beans in scientific notation.

21. Jack works part time with his dad installing carpet. They need to install carpet in a square room that has an area of about 876 square feet. Carpet can only be ordered in whole square yards.
   a. About how many feet long is the room?
   b. About how many square yards of carpet do Jack and his dad need in order to cover the floor of the room? Explain your reasoning.

**Extended Response**

22. Marissa’s cat is stuck in a tree. The cat is on a branch 23 feet from the ground. Marissa is 5.5 feet tall, and she owns a 16-foot ladder.
   a. Create a table that shows how high up on the tree the top of the ladder will reach if Marissa places the base of the ladder 1 foot, 2 feet, 3 feet, 4 feet, and 5 feet from the tree.
   b. How high will Marissa be if she places the base of the ladder the distances from the tree in part a and stands on the rung 2.5-feet from the top of the ladder?
   c. Do you think Marissa can use this ladder to reach her cat? Explain your reasoning.
The Ohio and Erie Canal

In 1825, work began on the historic Ohio & Erie Canal, a waterway that connected the cities of Cleveland and Portsmouth. By 1832, traffic flowed along the entire 308-mile route. The Ohio & Erie is no longer a working canal, but its grassy towpath remains a popular destination for joggers and cyclists.

Choose one or more strategies to solve each problem. For Problems 1–3, use the graph.

1. A canal boat began at Frazee House, 14 miles south of Cleveland, and traveled south along the canal. The graph shows the boat’s distance from Cleveland. At this rate, how many miles would the boat have been from Cleveland after 10 hours?

2. The boat traveled at the legal speed limit for the canal. What was the speed limit in miles per hour?

3. The canal used two types of locks to raise and lower the boats: lift locks and guard locks. There were 153 locks along the canal, and there were 139 more lift locks than guard locks. How many guard locks were there?
The Glenn Research Center

The Glenn Research Center, in northern Ohio, is one of NASA’s key research facilities. The technologies developed at the Glenn Research Center have made it possible for humans to walk on the Moon, receive photographs from Mars, and explore the outer reaches of our solar system.

Choose one or more strategies to solve each problem.

1. The Flight Research Building is an enormous hangar that can hold several aircraft at the same time. The base of the hangar is a rectangle measuring 250 feet by 65 feet. To the nearest foot, what is the length of the longest pole that can be stored on the floor of the hangar?

2. The center’s supersonic wind tunnel can produce wind speeds of up to 2280 mi/h. Here, scientists can test the effects of doubling wind speed. If scientists begin with a wind speed of $2^5$ mi/h. How many times can they double the speed and still stay within the wind tunnel’s capabilities?

For Problem 3, use the table.

3. The table shows some of the famous space missions that involved the Glenn Research Center. Use the following information to determine the destination of each mission.

• One mission went to Saturn, and one went to Mars.
• The shortest mission was a mission to the Moon.
• Saturn is farther from Earth than Mars.

What was the destination of the Pathfinder mission? the Apollo mission? the Cassini mission?

<table>
<thead>
<tr>
<th>Glenn Research Center Missions</th>
<th>Name</th>
<th>Distance to Destination (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pathfinder</td>
<td>$4 \times 10^7$</td>
<td></td>
</tr>
<tr>
<td>Apollo</td>
<td>$2.4 \times 10^5$</td>
<td></td>
</tr>
<tr>
<td>Cassini</td>
<td>$2 \times 10^9$</td>
<td></td>
</tr>
</tbody>
</table>